● 大井 雅雄 特定助教

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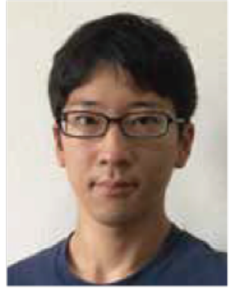

捻られた調和解析による Langlands 関手性の研究

私の専門は整数論という，その名の通り整数にまつわる現象を考察する分野です。現代の整数論はきわめて多様な方向に分化していますが，その中で私はLanglands 対応（予想）と呼ばれる仮説に興味を抱いています。数学ではしばしば，一見無関係な二つの異質な対象の間に不思議な関係性が見つかることがあります。Langlands 対応とは，整数論におけるこの類の関係性の中で最も重要なものの一つであると言えます。そしてこの仮説から存在が示唆される様々な未知の現象は，Langlands 関手性と総称されています。

私の白眉プロジェクトの研究では，調和解析と呼ばれる群上の解析学の理論を駆使することで，Langlands 予想を論理的に仮定せず Langlands 関手性を確立することを目指します。更には関手性から元の対応を逆算するかたちで，Langlands 予想そのものの証明にも迫りたいと考えています。

Study of the Langlands functoriality viatwisted harmonic analysis

In number theory, we study phenomena arising from integers. Among many directions in modern number theory, I am interested in the conjectural Langlands correspondence. In mathematics, we can often find a mysterious relationship between two completely different objects. We can say that the Langlands correspondence is one of the most important conjectures which predict such a kind of mysterious relationship in number theory. If we believe this conjecture, we can find a lot of unknown phenomena, which are called the Langlands functorialities. In my study in the Hakubi project, I will try to prove those Langlands functorialities by using harmonic analysis, which is a theory of investigating functions on groups. The point here is that this approach is logically independent of the conjectural Langlands correspondence. Thus I hope this study will eventually lead us to establish the Langlands correspondence by using the Langlands functoriality conversely.

What is number theory?

The objective of number theory is, as its name suggests, to investigate the properties of "numbers" such as integers. In fact, most of properties of integers (especially, prime numbers) are still veiled in mystery. Even at present, there are so many kinds of conjectures which are very elementary (i.e., you can understand what it says if you just know the definition of a prime number)but still unsolved. With such a kind of motivation originating in our primitive curiosity, modern number theory has been developing in numerous directions. Among them, I have been studying a hypothesis called the "Langlands conjecture". Here let us recall a famous classical conjecture proposed by Fermat. It claims that, for any integer n greater than 2, we do not have a triple (x, y, z) of nonzero integers

Fig.1 A bench where I often did some matrix computation together with a lot of mosquitos

satisfying the equation "$x^n + y^n = z^n$". Although its assertion is very elementary and simple, it took 350 years to be solved (now it is called Fermat's last theorem). The point here is that this conjecture was solved due to the development of modern number theory rather than just one flash of inspiration. In fact, we can understand Fermat's last theorem as one of very special cases of the Langlands conjecture.

Symmetry of numbers; Galois representations

One fundamental way in modern mathematics is to note the "symmetry" of a given mathematical object in order to investigate the properties of the object. Such a "symmetry" 2 can be mathematically formulated by using the notion of a "group". When we want to study such a group, we often consider a space (more strictly speaking, a vector space) which realizes the symmetry of the group. Such a space realizing the given symmetry is called a "representation" of the group. In general, there are many ways to take such a representation. The philosophy here is that we can understand the group and the original mathematical object by investigating all possible representations completely.

In our context of modern number theory, a mathematical object is nothing but the set of whole integers or rational numbers. Then its symmetry is called the "Galois group" and a representation of the Galois group is called a "Galois representation". We can translate many elementary problems on numbers such as Fermat's last theorem in this sophisticated language of Galois representations.

Langlands correspondence and Langlands functoriality

Roughly speaking, the Langlands conjecture predicts a natural connection (called the "Langlands correspondence") between Galois representations and other representations (called "automorphic representations") of a totally different group. If we have such a correspondence, it will be possible to understand a problem on Galois representations as that on automorphic representations. Then we can attack the problem from a totally different viewpoint by using tools developed in the world of automorphic representations. Indeed, the proof of Fermat's last theorem was carried out in this strategy after proving a special (but enough to solve the last theorem) case of the Langlands conjecture. Thus we can say that the Langlands conjecture is a very deep and big conjecture.

If we can establish the Langlands correspondence, we can apply it to so many problems. However, the perspective of my study is in a slightly different point. The point is that even if we are still far from establishing the Langlands correspondence, it often helps us to find new phenomena on automorphic or Galois representations. For example, for a given Galois representation, we can attach a new Galois representation to it by linear algebra. Thus if we believe the Langlands conjecture, there should be a corresponding way of constructing an automorphic representation from a given one. An important point here is that it should be very difficult to find this phenomenon on automorphic representations without knowledge of the Langlands correspondence. These kinds of phenomena are called the Langlands functoriality. The final goal of my project is to establish the Langlands functoriality without using (logically) the Langlands correspondence and to deduce the correspondence from the functoriality conversely.

Method: twisted harmonic analysis

I am going to use the theory of "harmonic analysis" to attack the Langlands functoriality. It is a tool to understand representations of groups. Especially, it enables us to transform an automorphic representation into a function on the group, which is called the "character" of the representation. Since an automorphic representation can be recovered from its character completely, we can interpret a problem of the Langlands functoriality in terms of characters. In some sense, a representation itself is too difficult to grasp because it is an abstract space (which is typically infinite dimensional!). However, in contrast, the character of an automorphic representation is much more concrete in a sense that it is just a function on the group. Thus I am planning to deepen a theory of analysis on the group ("twisted harmonic analysis") first, and then try to establish the Langlands functoriality by using it.

[Bibliography]

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